

# TRIANGULATION

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## CHAPTER I.

### ELEMENTARY PRINCIPLES.

Triangulation is a term which has in recent years been applied to certain operations in Sheet Metal Pattern Development, although said operations have long been explained in works upon Descriptive Geometry, where is found the declaration that the true length of a right line in space may always be found in the hypotenuse of a right angled triangle, whose base is equal in length to the horizontal projection of the line, and whose perpendicular is equal to the difference in length of the vertical projectors from the extremities of that line.

Triangulation as applied to sheet metal pattern development, is the act or process of dividing into triangles, also the results thus secured; specifically, the laying out and accurate measurement of a network of triangles presumed to be upon the surface of the object, and shown upon its geometrical representation which has been correctly delineated.

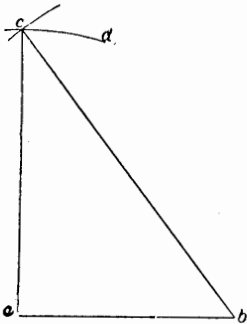
Triangles with which we deal are considered as plane triangles, although not strictly so, since a plane triangle is presumed to lie in one plane, and is bounded by three right lines. A triangle which is presumed to be a portion of the curved surface of an object will not lie in one plane. In many instances one side at least of said triangle is not a right line but a curved one. Thus many triangles involved in triangulation as applied to sheet

metal pattern development are mixtilinear triangles. However, the magnitude of the variation is so small, that it may consistently be considered as a negligible quantity.

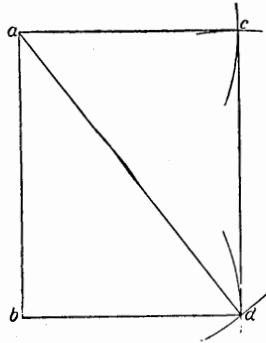
### TO DRAW A TRIANGLE.

As an aid in securing a clear conception of the most elementary principles involved, we may for the moment presume that we have given us three, four and five inches as the lengths of three sides of a triangle.

By the use of our compasses and straight edge, we are enabled to draw such a triangle by first drawing a line three inches long, as illustrated at *a b*, Fig. 1.



*Fig. 1. Illustrating a method of drawing a triangle to given dimensions.*



*Fig. 2. Illustrating a method of drawing a parallelogram when the length of its diagonal is known.*

With compasses set to a span of four inches, place one point at *a*, and describe the arc *c d*. Since the length of one side of the required triangle is four inches, the vertex of one angle must lie in the arc *c d*, and as the third side is required to have a length of five inches, the compasses may be adjusted to a span of five inches, and with point *b* as center, we may describe the small arc as at *c*,

thus locating the vertex of the third angle in its correct relative position. Lines may be drawn connecting points as shown in Fig. 1, thus forming the three sides of the required triangle. Here  $a b$  is the base,  $a c$  the perpendicular, and  $b c$  the hypotenuse.

#### TO DRAW A PARALLELOGRAM.

To draw a figure which is known as a parallelogram, to given dimensions, the magnitude of at least one angle, or the length of its diagonal, must be known.

As for example, we have given us the lengths of two parallel sides as four inches, and the distance between the extremities of these lines as three inches. We draw a line whose length is four inches, as illustrated at  $a b$ , Fig. 2.

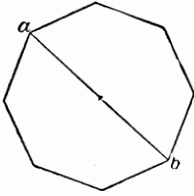
Since the extremities of the line forming the opposite side are to be three inches from points  $a$  and  $b$ , we may adjust our compasses to a span of three inches, and with points  $a$  and  $b$  as centers, describe arcs as shown at  $c$  and  $d$ . As these arcs have been drawn with a radius of three inches, every point of which they are composed must be three inches distant from their respective centers  $a$  and  $b$ . Therefore to locate the line which forms the second four inch side, the magnitude of at least one angle, or as above stated, the length of the diagonal must be known. Presuming this to be five inches, we adjust our compasses to a span of five inches, and with point  $a$  or  $b$  as center, describe a small arc intersecting the first as at  $d$ , thereby locating the vertex of the angle as at  $d$ , in its correct relative position. Since the side  $c d$  is known to be four inches long, point  $c$  is located as shown; lines are now drawn to complete the required figure.

Thus as will be noted, the parallelogram has been

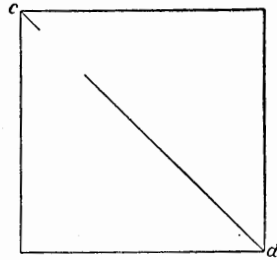
drawn by knowing the lengths of three sides of one of the two triangles of which it is composed.

### SOME SUGGESTIONS.

The student is advised to cut from sheet metal or cardboard two pieces, the forms of which are shown at Figs. 3 and 4. These forms may be looked upon as the forms of the top and base of an object which transforms from square to octagon, the square in this example being considered as the base.



*Fig. 3. Illustrating a form to be cut from sheet metal or cardboard.*



*Fig. 4. Illustrating a form to be cut from sheet metal or cardboard.*

Something for a center support as a block of wood whose ends have been made parallel, may be secured, and of a length suitable for the vertical height of an object whose size of base and top have previously been established in the pieces spoken of, and shown in Figs. 3 and 4. These pieces may be fastened to the block of wood, and so arranged that lines  $ab$ , and  $cd$ , as shown at Figs. 3 and 4, will be parallel; with the center of the top directly above the center of the base, as illustrated at Fig. 5.

We now have a form about which a flexible but non-elastic material (paper) may be formed, which if marked or trimmed at top and base, will when removed,

supply a pattern for an object whose dimensions have been established in said form, the surface of which may be looked upon as being composed of triangles. The lengths of sides of the square and octagon furnish the length of one side of each triangle. The lengths of the remaining two sides of each triangle in this example, will be found in the distances points *a* and *b* of the top are from point *d* of the base, or, in the true lengths of lines *a d* and *b d*, Fig. 6.

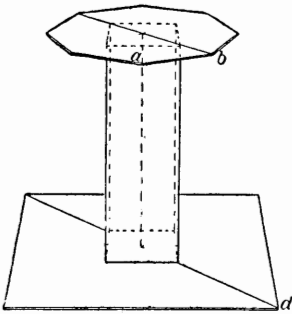


Fig. 5. Illustrating the relative positions the two pieces of sheet metal or cardboard should occupy.

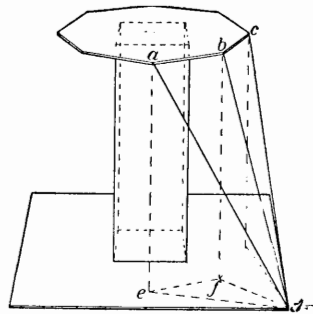


Fig. 6. Illustrating triangles from which the true lengths of lines may be secured.

It may be here remarked that in this example, it is only necessary to determine the lengths of two lines which represent the sides of one triangle, since there are in reality, but two lengths of side in the triangles forming the whole surface of the pattern, as will be hereinafter shown. If lines be dropped from points *a* and *b*, as shown at *a e*, and *b f*, Fig. 6, which are perpendicular to the top and base, lines may be drawn as *e d*, and *f d*. We then have in the true lengths of lines *a e* and *b f*, the perpendiculars of right angled triangles, the bases of which are *e d* and *f d*, when, as is clearly shown by Fig. 6, the true length of each line is found in the hypotenuse of its respective triangle.

From what has been stated above, the student will note that with the form previously constructed as shown at Fig. 5, he could drop imaginary lines as illustrated at Fig. 7, from the vertex of each angle of the octagon to the base, as in points  $g h i j k o n$  and  $m$ , and if lines be drawn upon the base to connect these points, he will have duplicated the form as previously established in the top.

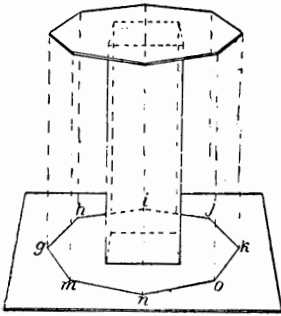


Fig. 7. Illustrating principles by which a plan is obtained.

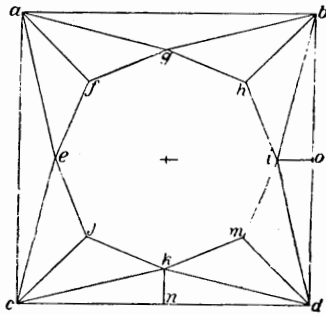


Fig. 8. A plan of the object.

Thus, as will be noted, the same results would have been secured, had we constructed a diagram as shown at Fig. 8.

#### A PLAN.

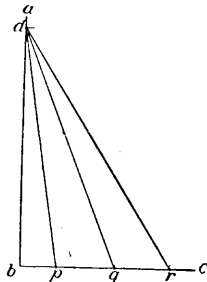
This diagram consists of the square  $a b c d$ , which is the exact form and size of the base, and is known as a plan of that portion of the object. The octagon  $e f g h i m k j$ , is the exact form and size of the top, and is known as a plan of that portion.

Since in orthographic projection, the intersection of two planes demands a line, we draw lines as shown at  $e c, j c, k c, k d$ , etc., Fig. 8, which represent in plan the sides of triangles of which the surface of the object is composed.

As the sides of the square and octagon supply the true lengths of one side of each triangle, we have simply to determine the true lengths of sides represented in lines  $k d$ ,  $m d$ , etc., to furnish all measurements necessary for completing a pattern. It has been previously shown that each of these lengths may be found in the hypotenuse of a right angled triangle, whose base is equal in length to the line in plan, and whose perpendicular is equal to the vertical height of the object.

#### DIAGRAM OF TRIANGLES.

In any convenient position we may draw lines as shown at  $a b$ , and  $b c$ , Fig. 9, which are at right angles



*Fig. 9. Diagram of triangles from which true lengths are obtained.*

to each other. Set off from  $b$  upon line  $a b$ , a distance equal to the vertical height of the object as at  $d$ . Upon referring to the plan Fig. 8, we find that two of the lines radiating from the vertex of each angle of the square are of equal lengths, and since the upper extremities of these lines are all at the same distance from the base, it follows that the true lengths of lines of which these are a plan, are equal in each group.

Upon measuring lines as  $k d$ , and  $i d$  of the plan, they are found to be of equal lengths, therefore in this ex-

ample there are but two lengths to be determined, unless for convenience, we assume two additional lines as  $k n$ , and  $i o$ , thereby enabling us to designate one quarter of the object in plan, as  $k m i o d n$ , which may be duplicated for the other three equal parts. Having previously located point  $d$ , Fig. 9, we may set off from point  $b$  upon line  $b c$ , distances equal to lengths of lines  $k n$ ,  $k d$ , and  $m d$ , as in points  $p q$  and  $r$ . Upon drawing lines as shown at Fig. 9, we have in the length of line  $d p$ , the true length of a line of which  $k n$  is a plan, in  $d r$  the true

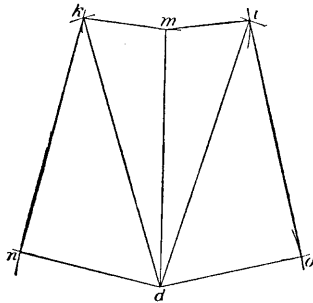


Fig. 10. Pattern for one-quarter of the object.

length of a line of which  $k d$  is a plan, and in  $d q$  the true length of a line of which  $m d$  is a plan.

We are now in a position to develop a pattern, one quarter of which is shown at Fig. 10, since the plan Fig. 8, and the diagram of triangles Fig. 9, supply all necessary measurements for that purpose.

#### THE PATTERN.

In any convenient position draw a line whose length is equal to length of line  $d q$  Fig. 9, as shown at  $m d$  Fig. 10.

It may be remarked that this line is in reality the line shown in perspective at  $b d$ , Fig. 6, and that there are

two additional lines radiating from point  $d$ , as shown in perspective by  $d a$ , and  $d c$ . The distances between the extremities of those lines at  $a$  and  $c$ , Fig. 6, are found in lengths of lines  $m k$ , and  $m i$ , Fig. 8. With compasses set to a span equal to the length of line  $d r$ , Fig. 9, and with point  $d$ , Fig. 10, as center, describe arcs as shown at  $k$  and  $i$ . With compasses set to a span equal to length of lines  $k m$  or  $m i$ , Fig. 8, and with point  $m$ , Fig. 10, as center, describe arcs as also shown at  $k$  and  $i$ , thereby locating those points in their correct relative positions.

Presuming that only one quarter of the pattern is to be developed as shown, and that the seam is required to be upon a line as shown at  $k n$  or  $i o$  of Fig. 8, there is an additional line radiating from points  $k$  and  $i$ , as shown in plan at Fig. 8. The true lengths of these lines have been found in  $d p$ , Fig. 9, therefore the compasses may be set to a span equal to the length of that line, and with points  $k$  and  $i$ , Fig. 10, as centers, the small arcs may be drawn as shown at  $n$  and  $o$ . Since the plan, Fig. 8, supplies the true length of one side of the triangles  $k n d$ , and  $i o d$ , the compasses may be set to a span equal to the length of line  $n d$  or  $d o$ , Fig. 8, and with point  $d$ , Fig. 10 as center, describe the small arcs cutting the first at points  $n$  and  $o$ , when lines may be drawn as shown, which completes the pattern for one quarter of the object.

This is all that is necessary, since it may be duplicated for the remaining three equal parts, or the lengths of lines as shown may be used in rotation to develop the whole pattern.