

PART III

GEOMETRY — PLANE AND SOLID

SINCE the study of pattern drafting is necessarily of a geometrical nature, an elementary knowledge at least of abstract geometry will be found necessary to its successful pursuit and it is almost unnecessary to add, that the better that science is understood by the beginner, the easier and more rapid will be his progress.

This is said in the hope that the student will avail himself of any opportunities he may have to advance himself along that line. The treatment contained herein includes only that which is considered as essential to the work in hand. This is one of the branches of science which, of all others, demands that its study should be pursued systematically. As in climbing a ladder one must begin at the bottom, so in the study of geometry each step depends upon the preceding one. It is a course in the logic of measurements in which nothing, be it ever so simple, is accepted until it is proven, except it be self-evident, in which case it is known as an *axiom*.

Geometry, which means literally the measurement of the earth, is said to have for its object the measurement of extension and has been called the science of space. It is the science which treats of the properties, relations and measurements of *lines*, *angles*, *surfaces* and *solids*; it being understood that lines and surfaces may be extended indefinitely in either direction. The above named quantities are termed geometrical forms or magnitudes.

For instance, if a block of stone or wood be cut so that it shall have six perfectly square faces, it would represent what in geometry is termed a cube; but the geometrical term cube refers only to the shape or surface of such a block and not to the block itself.

A mark drawn by a sharp instrument upon paper or upon any surface is only the representative of a line, the line itself being the distance between the two points at the extremes of the mark following the course of the mark be it straight, broken or curved. The word line as applied to a mark is really erroneous but is tolerated as a matter of convenience. Terms of this character refer entirely to forms which are said to exist in *space*, and care must be taken by the student to distinguish between

the abstract forms so designated and any concrete or physical forms which might be supposed to be described by these terms or by the names of any geometrical forms classified under these heads.

In place of the word *solid* some authors have substituted the word *volume* because the former word seems to carry with it the idea of material substance, something which can be weighed or cut.

While the subject matter of geometry is the means by which final results are obtained in many fields of applied science, it has another very important value which should not be overlooked. It is sometimes characterized as the essence of logic and as such, when properly and systematically studied it probably does more to discipline the mind, to lead it into a systematic and orderly way of thinking than any other branch of science. It teaches exactness in the use of terms, the most concise statement of facts and the elimination of all that is superfluous. Its course of reasoning is the most direct road to a conclusion based upon the facts given. It teaches one to systematize his mind, to properly connect and store away as it were, all the fact of any given subject, just as a business man files his letters and papers, so that when wanted they may be easily found. To know how to think is worth more than it is to know how to solve any individual problem, because a systematic way of thinking will lead to the solution of all problems.

The practical work of geometry will be taken up in the solution of the problems which will follow.

Geometrical Problems

In the solution of geometrical problems there are usually two and sometimes several methods of solution. One method is by means of the T square and triangles, which may be termed the mechanical method. The other and more strictly geometrical method is by means of the compass and straight edge only. The latter method will be of special interest to pattern cutters from the fact that it is often required to perform these operations upon the metal. In some shops, drafting paper is not extensively used. We shall, therefore, make use of the geo-

metrical methods first and treat of the others afterward.

1. *To Draw a Straight Line Parallel to a Given Line and at a Given Distance from it.*— In Fig. 110, let A B be the given straight line. From any two points on this line, as c and a, describe arcs by means of the compasses, with a radius equal to the given distance, as at a a and b b. Bring the straight edge tangent to these two arcs and draw the required line as shown by C D.

given line and C the point from which it is desired to let fall a perpendicular. With C as center and any radius greater than the distance from C to the line A B, describe an arc cutting A B at points a and b as shown. Then from a and b as centers, with any convenient radius greater than one-half of a b, describe two arcs intersecting each other at c. A line drawn from C toward c and ending at line A B will be the desired perpendicular.

4. *To erect a Perpendicular Near the End of a*

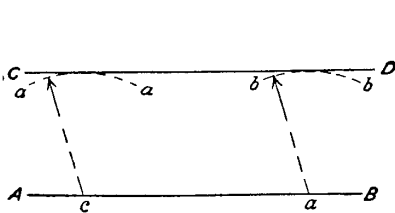


Fig. 110

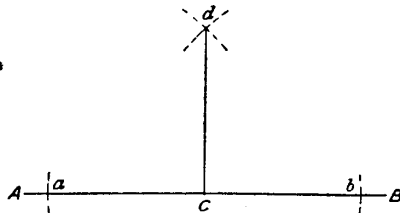


Fig. 111

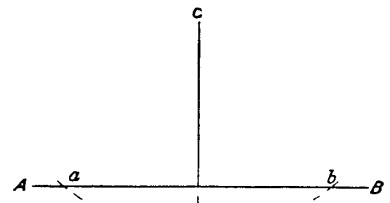


Fig. 112

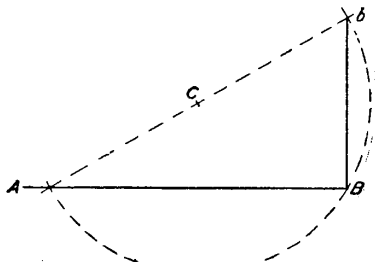


Fig. 113

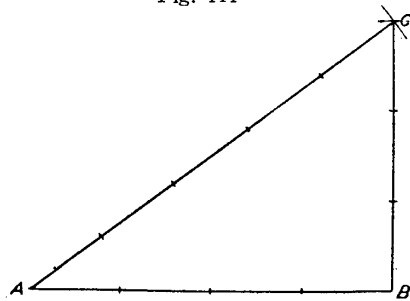


Fig. 114

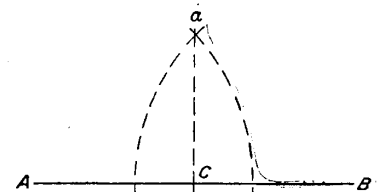


Fig. 115

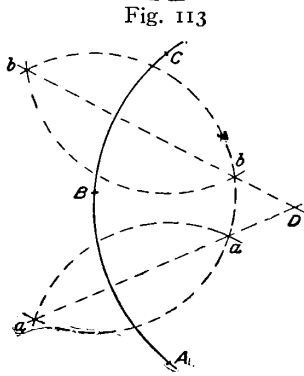


Fig. 116

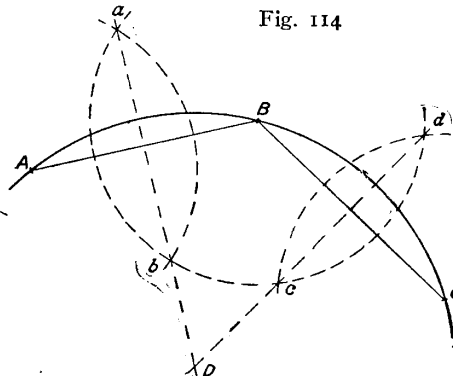


Fig. 117

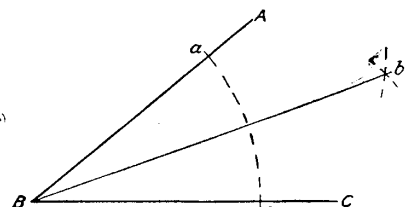


Fig. 118

2. *To Erect a Perpendicular to a Given Line at a Given Point Within the Line.*— Let A B of Fig. 111 be the given line and C the given point. From C as a center, with any convenient radius, describe short arcs cutting the line A B, as shown at a and b. Then from a and b as centers, with any radius greater than a C, describe arcs intersecting each other, as shown at d. A line drawn from d to C will then be perpendicular to the line A B at C.

Given Line.— Let A B in Fig. 113 be the given line, near one end of which, as at B, it is desired to erect a perpendicular. From any convenient point, as C, without the line A B, describe a circle whose radius is C B, extending it in one direction to cut the line A B and in the other direction indefinitely. Draw a line from the point of intersection through C extending it to cut the arc at b. Then a line drawn from b to B will be the desired perpendicular.

3. *From Any Given Point Without a Line to Drop a Perpendicular.*— Let A B, Fig. 112, be the

Another Method.— It is a commonly known fact that in any right-angled triangle whose base is

equivalent to four parts and whose altitude is equal to three, that the hypotenuse is equal to five parts; therefore make AB of Fig. 114 equal to four units of any denomination. Then from B as a center with a radius equal to three of the same units, describe a short arc near C , and from A as center, with a radius equal to five of the units, describe another arc, cutting the first arc as shown at C and draw CB . Then will the angle at B be a right angle and BC will be perpendicular to BA .

5. *To Bisect a Straight Line.*—Let AB in Fig. 115 be the line which it is desired to bisect. From the ends A and B , with any radius greater than one-half of AB , describe arcs above and below the line intersecting each other as shown at a and b . A line drawn from a to b will bisect the line AB at C .

6. *To Find the Center of a Given Arc.*—Let ABC of Fig. 116 be the given arc. From any point near the middle of the arc, as B , with any convenient radius describe an arc indefinitely as bb as shown. From A and C as centers, with the same radius, draw two arcs cutting the first arc as shown at aa and bb . A line drawn through aa extended to meet a line drawn through bb , as shown at D will give the center from which the arc was struck.

7. *To Draw a Circle Through Any Three Given Points Not in a Straight Line.*—Let A, B and C , in Fig. 117, be three points through which it is required to draw an arc of a circle. Connect A with B and B with C , then bisect these lines according to the problem shown in Fig. 115, extending the lines to meet, as shown at D . Then from D as a center, a circle may be drawn which will pass through the given points.

8. *To bisect a given angle.*—Let ABC of Fig. 118 be the given angle. From B , the vertex of the angle, with any convenient radius, cut the sides of the angle as shown at a and c . Now from these points as center, with any radius greater than one-half of ac , describe arcs intersecting each other at b . A line drawn from B to b will bisect the angle.

9. *To Divide a Straight Line into Any Proposed Number of Equal Parts.* (7 in this case.)—Let AB in Fig. 119 be the line which is to be divided into seven equal spaces. From B draw the straight line BC forming any angle with BA . Set off on BC seven equal parts of any length. Join the point 7 to A and parallel to this line from the points 1, 2, 3, 4, 5 and 6, draw lines cutting the line AB as shown, which will divide this line into the desired number of parts.

10. *To Transfer Any Angle.*—Let aCb in Fig. 120 be the given angle to be transferred to the line AB . Using C as center with the desired radius,

draw the arc cutting the sides of the angle at a and b . Using the same radius, with B as center draw similar arc cutting the line AB at a' and extend the other end of the arc indefinitely. Now take the distance from a to b and set it off as shown from a' to b' . Draw a line from B through b' which will be a reproduction of the given angle.

11. *To Inscribe a Circle in a Given Triangle.*—Let ABC in Fig. 121 be the given triangle. Bisect any of the two angles as explained in connection with Fig. 118, in this case at the angles at A and B

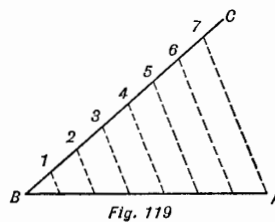


Fig. 119

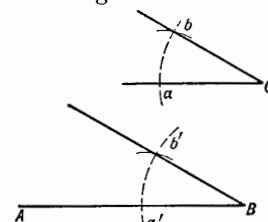


Fig. 120

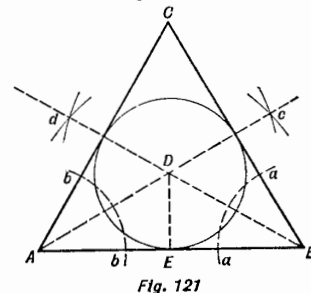


Fig. 121

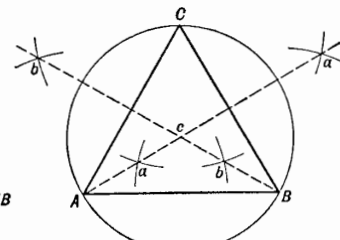


Fig. 122

in Fig. 121. Where these two lines Ac and Bd intersect at D , drop a perpendicular line cutting the line AB at E . Using D as center with DE as radius, draw the circle shown, which will touch the three sides of the triangle.

12. *To Describe a Circle About a Given Triangle.*—Let ABC in Fig. 122 be the given triangle. Bisect any of the two sides as explained in Fig. 115 and where the two bisecting lines aa and bb in Fig. 122 intersect at c , use this as a center, and with cA as radius describe the circle shown.

Geometrical problems can also be solved by the use of the compasses and straight edge only and by means of the T square and triangles. On account of the almost universal use of these instruments and of the ease with which the work can be done by their use, it is of greater importance that this method should be mastered and used in the construction of the regular polygons, following this with the more strictly mathematical methods, for use when circumstances require them.

Construction of Regular Polygons

To Construct a Square.—Placing the head of the T square against the left side of the drawing board,

slide it along to the required position and draw a horizontal line along its edge, as A D of Fig. 123, making it the desired length. Placing one of the triangles against the blade of the T square, bring one of its perpendicular sides to the points A and D and draw perpendiculars of indefinite length. The height of one of the perpendiculars, as D C, can be determined by drawing the arc A C from D as a center, as shown, or by bringing the oblique side of the 45° triangle to the point A, and drawing the diagonal line to intersect D C at C, after which a horizontal line drawn from C to meet a vertical line drawn from A, at B, will complete the square.

In solving this problem geometrically, the perpendicular D C can be erected by the method shown in either Fig. 113 or Fig. 114 and the point C, in Fig.

may be constructed first. Then with half the diagonal, as A x of Fig. 126 as a radius, and the points A, B, C and D as centers, draw four arcs intersecting the sides of the square at c and h, b and e, d and g, and finally, f and a, when the points a and b, c and d, etc., may be connected by straight lines, as shown, to complete the octagon and the lines A b, A a, etc., erased.

Should it be required to construct an octagon when the length of one side only is given, first draw A H, of Fig. 125 to the required length, and from its central point a, erect a perpendicular of indefinite length; then, with the triangle in the position shown at b, draw the side H G, making it equal in length to A H, and from its central point b draw a perpendicular to H G (by reversing the position of the

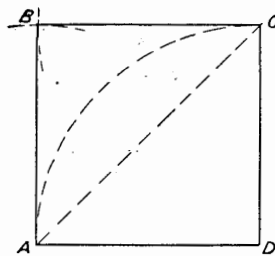


Fig. 123

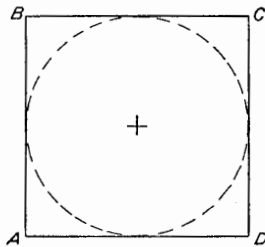


Fig. 124

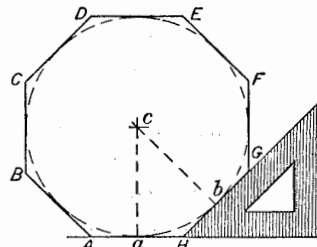


Fig. 125

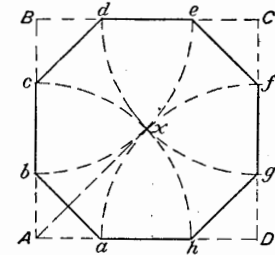


Fig. 126

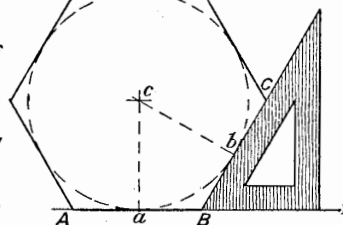


Fig. 127

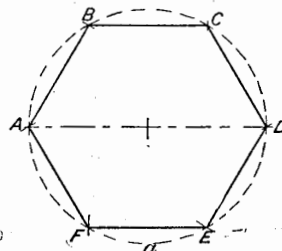


Fig. 128

123 located by an arc whose radius is A D as before, after which, using A and C as centers, two arcs may be drawn with the same radius to intersect at B, and A B and then B C drawn to complete the figure.

A convenient method for drawing a square is to first draw a circle whose diameter is equal to the required length of one side, after which the four sides may be drawn tangent thereto, extending them to meet at the angles as shown in Fig. 124.

To Construct a Regular Octagon.—The simplest method of constructing an octagon is to first draw a circle of the required diameter, as shown in Fig. 125, and to then draw the oblique sides by bringing the oblique side of the 45° triangle tangent to the circle in four positions, one of which is indicated at b; adjacent sides being continued to intersect as shown at A, B, C, etc.

According to the geometrical method, the square

triangle) and continue it to intersect the first perpendicular, as shown at c, which will be the center of the inscribed circle. From c, as center with the radius c a, draw the inscribed circle, and complete the figure by drawing the remaining sides tangent, as previously described.

To Construct a Hexagon.

—A hexagon may be constructed or circumscribed about a circle in the same manner as explained in the case of an octagon, but using the 30° × 60° triangle instead of the 45°, as shown in Fig. 127. This makes the diameter of the hexagon, as measured through the center of opposite sides, equal to that of the circle.

If it is desired to construct a hexagon of stated diameter as measured through the angles the geometrical method will be most suitable: First, draw a circle to the stated diameter, then, using the radius as a chord, start at any desired point, as A, of Fig. 128, and space or step around the circle locating the points B, C, D, E and F, which connect by straight lines as shown. The hexagon may be so placed as to have two of its sides vertical by beginning the spacing at a.

If it be required to construct a hexagon when the length of one side only is given, a method similar to that given for the octagon may be employed: Draw A B and B C of Fig. 127 to the required length by means of the T square and the 30° by 60° triangle, then bisect these sides and draw the per-

pendiculars $a c$ and $b c$ as before, meeting in c , which is the center of the inscribed circle. The remaining sides are drawn tangent to the circle as before described.

This same problem can also be solved by strictly geometrical means, as shown in Fig. 129, by taking the length of the given side as a radius and describing arcs from A and B , the ends of the line, extending them to meet as shown at C , which point is the center of the circumscribing circle, when the figure

$B C$, which make equal to $A B$. Now, bisect the two sides, $A B$ and $B C$, and erect the perpendiculars, meeting at D as shown. From D as center, with $D B$ as radius, describe the circumscribing circle, and, using $A B$ as a chord, space off the remaining sides.

The use of the protractor is, of course, a mechanical means. It may be remarked that there is no strictly geometrical method of constructing regular figures having seven, nine, eleven, etc., sides.

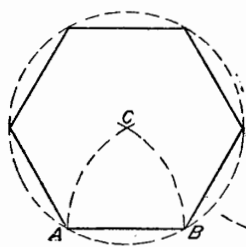


Fig. 129

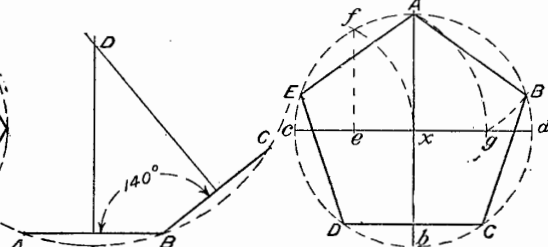


Fig. 130

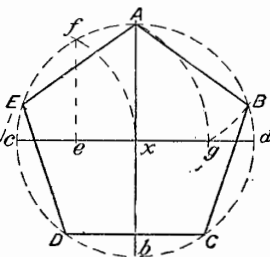


Fig. 131

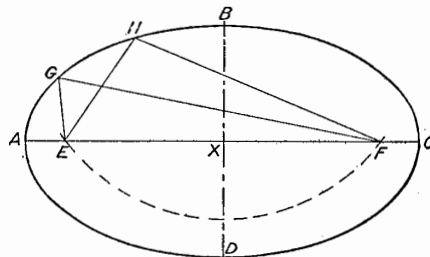


Fig. 132

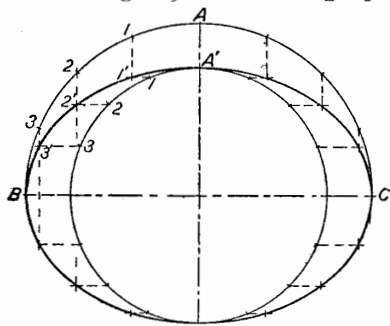


Fig. 133

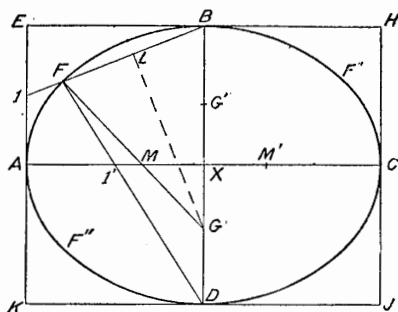


Fig. 134

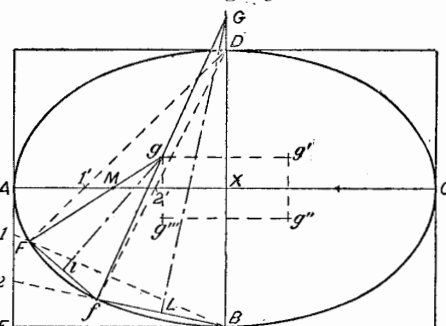


Fig. 135

Construction of Polygons and Ellipses.

may be completed as explained in connection with Fig. 128.

The method shown in Figs. 125 and 127 can be applied to the construction, upon a given side, of a polygon having any number of sides, when the angle between two adjacent sides is known. By means, therefore, of a formula for finding the angle between the adjacent sides of any regular polygon, this method becomes a general rule and of universal application, and may be used to advantage in cases wherein the angles cannot be obtained by the draftsman's triangles. The angle, in degrees, between the adjacent sides of any regular polygon may be obtained by dividing 360 by the number of sides in the required polygon, and subtracting the result from 180. Thus, for a nonagon (nine-sided figure) $360 \div 9 = 40$ and $180 - 40 = 140$.

To apply the rule, first draw $A B$ of Fig. 130 the length of the required side; then, placing the center of the protractor at B , and one of its sides along the line $A B$, set off 140 degrees from left to right, and through the point so obtained draw the line

To Construct a Pentagon.—A five-sided figure can be constructed within a circle in the following manner: First, divide the circle into quarters by the vertical and horizontal diameters $A b$ and $c d$ of Fig. 131. Bisect the radius $c x$ by any convenient means, obtaining the point e . From e as center, with $e A$ as radius, describe the arc cutting the diameter $c d$ at g ; then $A g$ will be the length of one side of the pentagon. With this distance between the points of the dividers, begin at any point, as A , and space off the circle, as shown by B, C, D and E , and draw the chords $A B, B C$, etc.

The construction of a pentagon upon a given side can be most easily accomplished by the application of the general rule given above.

Polygons of *ten, twelve* and *sixteen sides* can be constructed upon the diagrams of the pentagon, hexagon and octagon by first bisecting the arcs subtended by the several sides in the manner shown in Fig. 115 thus obtaining the intermediate points on the circumscribing circle, necessary to complete the figure.

The Ellipse

An ellipse is described as a plane figure bounded by a curve such that the sum of the distances from any point thereon to two points within the figure, called the foci, is the same as that of any other point. While this definition accurately describes the nature of the curve, it conveys no idea of its shape to those unfamiliar with it, until it is shown by the use of a diagram that the statement made with regard to the sum of certain distances is true. Strange as it may seem, however, the use of an incorrect expression — that is, to speak of an ellipse as an elongated circle — conveys an idea of its shape. Owing to its elongated shape it has a long and a short diameter, which are spoken of as the major and minor axes, which are necessarily at right angles to each other. In Fig. 132 A B C D shows an ellipse, of which A C is the major and B D the minor axis, the points E and F being what are termed the foci. According to the definition given above, the sum of the distances from any point on the curve, as G, to E and F is equal to the sum of the distance from any other point, as H, to E and F, or, in other words, $GE + GF$ is equal to $HE + HF$.

Applying this statement now to points A and B we have $BE + BF$ is equal to $AE + AF$ (or, in other words, equal to A C, because $FC = AE$). From this it is apparent that BE, which is one-half of $BE + BF$, is equal to AX, which is one-half of A C. Therefore the positions of the foci in any ellipse may be found by taking one half of the major axis A C as a radius and using B or D, the extremity of the minor axis, as a center, describing an arc to cut the major axis, as shown at E and F.

Having found the foci, the elliptical curve may be described mechanically by the following method: Fix two pins firmly at points E and F, placing a third at B. Fasten a cord to the pin at one focus, as F, pass it around the pin at B and secure it to the pin E, drawing it so tightly that there shall be no slack; then, substituting a pencil point for the pin B, pass it along in either direction, keeping the string taut, thus describing the ellipse.

The ellipse is often erroneously spoken of as an "oval." Though the word "oval" is generally accepted as meaning an ellipse, it really means an egg-shaped curve, which, by analogy, would signify a curve which is semicircular or nearly so at one end, while it is more pointed, or semi-elliptical, at the other.

There are several geometrical methods of describing an ellipse. A very available method is shown in Fig. 133, which consists in first drawing two cir-

cles from the same center, whose diameters are respectively equal to the major and minor axes of the required ellipse. Having divided both circles into the same number of equal spaces, beginning at corresponding points in each, as at A and A', and as shown by the points 1, 2 and 3 on each, draw lines outward from points on the inner circle, parallel for instance, to the diameter B C, which intersect by lines from points of corresponding number on the outer circle, drawn inward and at right angles to the first, as shown at 1', 2' and 3', through which trace the curve as shown from B to A'.

The ellipse is principally important to the sheet metal worker when used as the outline of an arch as well as for flaring articles. Should it be required to develop the patterns of articles of elliptical shape by the method of triangulation, a true ellipse may be used, but when it becomes necessary to construct moldings to its shape it is much more convenient to use instead an approximation to an ellipse composed of arcs of circles, and the same is true in the case of some flaring articles.

In regard to the character of the ellipse, it should be understood that through the four points, as A, B, C and D of Fig. 132 only one ellipse can be drawn. A fuller curve could, of course, be drawn from A to B, which would pass outside of the points G and H, or a scunter curve, which would pass inside of those points, but neither would be a true ellipse, because it would not conform to the measurements previously described.

The simplest form of an approximate ellipse is that composed of arcs of two circles, and for the construction of such a curve there are many methods. The method shown in Fig. 134, however, gives the most satisfactory result, since, by its construction, the point F, where the two arcs meet, is on the line of a true ellipse, which cannot be said of many other methods.

To construct an approximate ellipse by this method, first draw the rectangle E H J K to the length and width of the required ellipse and bisect it by the lines A C and B D, which become the major and minor axes. Divide A E and A X each into two equal parts, as indicated by the points 1 and 1'. Connect point 1 with B and from D draw a line through 1' and extend it to meet the line 1 B, as shown at F. Bisect F B, obtaining the point L, and draw L G at right angles to F B, extending it to meet B D as shown at G, which will be the center for drawing the arc F B. Now draw G F, intersecting A X at M, which will be the center for the arc F A. The arc F B is extended to F', while F A is extended to F". The centers for complet-

ing the figure are at G' and M' , and are found by making $X G'$ equal to $X G$ and $X M'$ equal to $X M$.

A closer approximation to a true ellipse can be made by constructing each quarter of arcs of three circles, as shown in Fig. 135. This method can be used to advantage in the matter of appearance in the case of a large elliptical arch. The method differs from that just explained only in the fact that the lines $A E$ and $A X$ are each divided into three equal parts, instead of two, as before, as shown by the points 1 2 and 1' 2'. Draw the lines 1 B and 2 B and from D draw lines through points 1' and 2' to meet 1 B and 2 B respectively, as shown at F and f . Bisect $f B$ at L and draw the perpendiculars as before, extending it to meet B D extended, which it does at G, which is the center from which to draw the arc $f B$. Now draw $G f$, which may, for convenience, be termed the last radius of this arc. Next, draw $f F$, bisect it at l and draw the perpendicular $l g$, which extends to meet the last radius $f G$, of the first arc, which it does at g , which becomes the center of the arc $f F$. Finally, draw the last radius $g F$ of this arc, intersecting A X at M, which point becomes the center of the third or last arc of this quarter. The centers G and M can be transferred to opposite points on the axes, as explained in Fig. 134, while the intermediate center, g , can be located in the other quarters of the figure, as

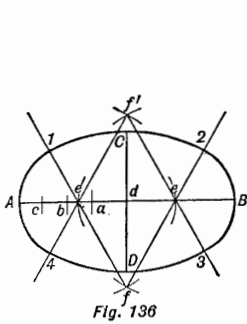


Fig. 136

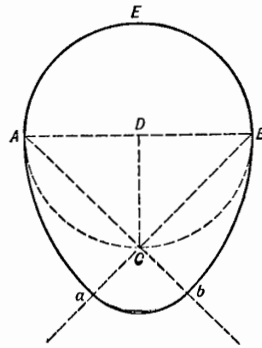


Fig. 137

shown at $g' g'' g'''$, by the construction of the small rectangle bearing those letters.

It is interesting to know that an ellipse is also an oblique section of a cylinder as well as of a right cone, and can also be drawn by the development of those sections: These operations will be considered at their proper places. Still another quick approximate method of drawing an ellipse is shown in Fig. 136.

Let A B represent the length of the ellipse and C D the width. Set off the width from B to a and divide $a A$ into three equal parts. Take two of these parts and set it off on either side of the

center d as shown from d to e . Now using e as center and $e e$ as radius, describe arcs intersecting each other at f and f' . Draw lines from f and f' through e indefinitely as shown by $f 1, f 2$, and $f' 3, f' 4$. Now using f and f' as center and $f C$ and $f' D$ as radius, describe the arcs 1-2 and 3-4 respectively. Now using e as center with radius equal to $e A$ and $e B$ complete the ellipse by drawing the arcs 1-4 and 2-3 respectively.

An egg shape oval may be drawn as shown in Fig. 137, where A B shows its width. Bisect A B and obtain D, which use as a center and complete the circle A E B C. From the center D draw a perpendicular cutting the circle at C. Draw lines indefinitely from B and A through C as shown. Using A and B as centers with radius equal to the diameter of the circle draw the arcs B b and A a . With C as center and C a as radius complete the figure by drawing the arc $a b$.

Although the list of geometrical problems here given might be extended almost without limit, we have deemed it expedient to confine ourselves to those of principal importance.

The one great problem or aggregate of all problems, as it were, in sheet metal work, is that of determining, or developing, if necessary, the shapes of the various faces of what appears to be a solid; and since the forms used in structural sheet metal work are usually regular or geometrical in character, every surface to be developed thus becomes a part of if not the entire surface of some geometrical solid. It will therefore be greatly to the advantage of the prospective pattern draftsman that he have a good knowledge of the general principles of solid geometry, or as it is sometimes termed, geometry of three dimensions.

In geometry a solid is defined as a portion of space bounded by planes. It has three dimensions, viz., length, breadth and thickness (height or depth). Although the term "solid" implies substance or material, it must not be overlooked that geometry treats only of the form.

The solids treated of in geometry are termed polyhedrons, a term signifying many sides.

When three or more planes intersect at a common point, the angle thus produced is termed a polyhedral or solid angle.

There are only five regular polyhedrons, viz.:

1. The *tetrahedron*, which is bounded by four equilateral triangles. This solid is also a triangular pyramid.
2. The *hexahedron* or cube, which is bounded by six squares.
3. The *octahedron*, which is bounded by eight

equilateral triangles. This solid is the same as though two square pyramids were placed together base to base.

4. The *dodecahedron*, which is bounded by twelve regular pentagons.

5. The *icosahedron*, which is bounded by twenty equilateral triangles.

The number and shape of the sides for each of these solids being known, it will be an interesting problem to lay out the pattern for any one of them so that it can be cut and formed from one piece of metal.

Aside from these, is a list of solids whose proper-

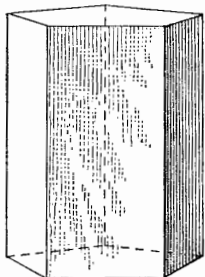


Fig. 138.—Prism

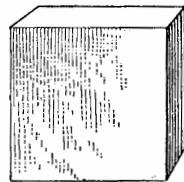


Fig. 140.—Cube

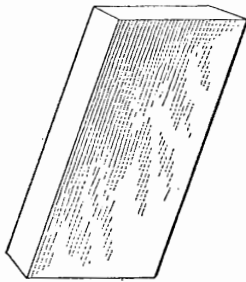


Fig. 139.—Oblique Prism

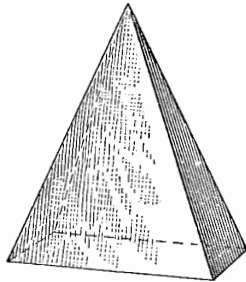


Fig. 141.—Pyramid

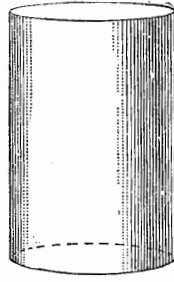


Fig. 142.—Cylinder

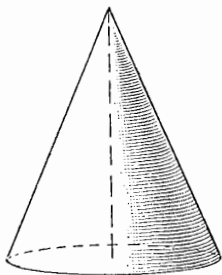


Fig. 143.—Right Cone

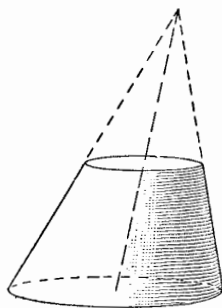


Fig. 144.—Frustum of Scalene Cone

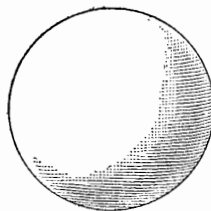


Fig. 145.—Sphere

Geometrical Solids

ties should be carefully studied and well understood. They include the *prism*, *pyramid*, *cylinder*, *cone* and *sphere*.

The prism is a solid whose bases are equal and parallel polygons, and whose sides are parallelograms, as in Fig. 138. Since the angles of a parallelogram are not necessarily right angles, it will thus

be seen that a prism may be oblique or scalene as well as right or erect, as in Fig. 139.

Prisms, cylinders and numerous irregular solids are said to have two bases, an upper and a lower, though in some cases they are neither equal nor parallel. This is especially true in the case of certain solids or forms with which the pattern draftsman must deal termed transition pieces, in which one base, as the lower, is required to fit a pipe of one shape, as a rectangle, while the other base takes the shape of another pipe, as an oblong.

The cube in Fig. 140 is a solid bounded by six equal squares. It is also a prism, since its bases are equal and parallel and its sides, being square, are therefore parallelograms. The usual idea of a prism is that of a solid having triangular bases, but its bases may have any number of sides.

A pyramid is a solid whose base is a polygon and whose sides are triangular as shown in Fig. 141. The vertices of the triangles forming the sides of the pyramid meet in a common point, forming the solid angle called the vertex or apex of the pyramid. Like the prism, its base may have any number of sides. The pyramid exists in a concrete form as in a hip roof.

An important difference between a prism and a pyramid is that in the one the dihedral angles forming its edges are parallel, while in the other they all converge to an apex.

In solid geometry curved surfaces are said to be generated by a straight line while being moved along under certain conditions or limitations.

A cylindrical surface is generated by a line moving along a fixed curve, being kept at all times parallel to a given line. In the case of the true cylinder the given line is its axis. If now instead of remaining parallel, one end of the generating line is held at a fixed point, while the other end traverses the curve, the surface generated is conical.

This idea can be illustrated practically by supposing that a stiff wire is used to represent the generating line and that, as it is moved in accordance with either

of the above described conditions, being held against a templet, it passes through a mass of wet clay or other plastic material. The surfaces thus cut or generated will be cylindrical or conical according to the condition adhered to. These explanations are introduced because they assist in conveying the strictly geometrical conception of the

cylinder and cone. It will thus be seen that the cylinder and the cone may have a base of any curved outline. This is important in view of the fact that all moldings have cylindrical surfaces, because their profiles, though not always circles, or arcs thereof, are still curved lines.

A right cylinder is a solid whose bases are circles and whose axis is perpendicular to its bases, as in Fig. 142. It is also termed a cylinder of revolution, because it is said to be generated by the revolutions of a rectangle about one of its sides as an axis. In such an operation, the moving side is thus always parallel to the fixed side or axis, while each of the other two sides of the rectangle becomes the radius describing the circle of a base representing the fixed curve.

A right cone may be generated in a similar manner by the oblique side of a right angled triangle when the triangle is revolved about one of its other sides, as in Fig. 143. A pyramid or cone may be oblique or scalene when its vertex is not over the center of its base; that is, when its axial line is oblique, as in Fig. 144.

In the generation of either the cylinder or the cone the generating line in any position is called an element of the surface. This is a most important point for the pattern draftsman to keep in mind, since it is upon the elements of a curved surface that all measurements are taken in developing its shape when the solid is cut off obliquely, as in the operations of miter cutting.

The frustum of a cone or pyramid is that part which remains after its upper part has been removed by a plane passed between the base and the vertex. Fig. 144 shows the frustum of a scalene cone. Strictly speaking, the cutting plane is parallel to the base, but since oblique sections of cones or pyramids are of quite frequent occurrence in sheet metal work, it is convenient, when the cone has been cut by an oblique plane, to speak of the remaining solid as an oblique frustum.

The sphere in Fig. 145 is a solid bounded by a surface every point of which is equidistant from a point within called the center. As a solid of revolution it is said to be generated by a semicircle revolved about its diameter as an axis.