MENSURATION.

Mensuration is that branch of arithmetic which is used in ascertaining the extension and solidity or capacity of bodies capable of being measured.

DEFINITIONS OF ARITHMETICAL SIGNS.

$=$ Sign of Equality, as $4 + 8 = 12$.

$+$ Sign of Addition, as $6 + 6 = 12$, the Sum.

$-$ Sign of Subtraction, as $6 - 3 = 3$, the Remainder.

$\times$ Sign of Multiplication, as $8 \times 4 = 32$, the Product.

$\div$ Sign of Division, as $24 \div 6 = 4$ $\frac{24}{6}$ $24 = 4$.

$\sqrt{}$ Sign of Square Root, signifies Evolution or Extraction of Square Root.

^{2} Sign of to be Squared, thus $8^2 = 8 \times 8 = 64$.

^{3} Sign of to be Cubed, thus $3^3 = 3 \times 3 \times 3 = 27$.

MENSURATION OF PLANE SURFACES.

To find the area of a circle—Fig. 52. Multiply the square of the diameter by .7854.

To find the circumference of a circle. Multiply the diameter by 3.1416.

Circle: Area $= .7854D^2$

Circ. $= 3.1416D$

To find the area of a semi-circle.—Fig. 52. Multiply the square of the diameter by .3927.

To find the circumference of a semi-circle. Multiply the diameter by 2.5708.

Semi-circle: Area = $.3927D^2$
Circ. = 2.5708D

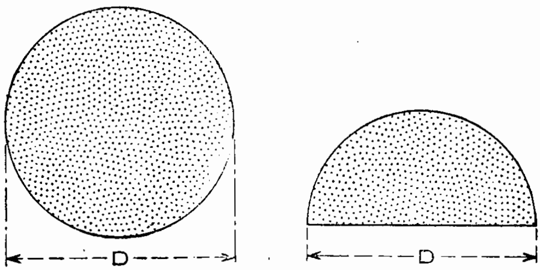

Fig. 52.

To find the area of an annular ring—Fig. 53. From the area of the outer circle subtract the area of the inner circle, the result will be the area of the annular ring.

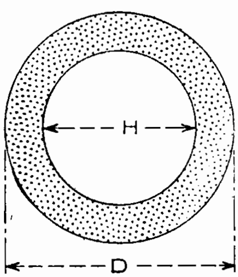

Fig. 53.

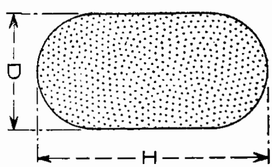

Fig. 54.

To find the outer circumference of an annular ring. Multiply the outer diameter by an 3.1416.

To find the inner circumference of an annular ring. Multiply the inner diameter by 3.1416.

Annular ring: Area = .7854 $(D^2—H^2)$
Out. circ. = 3.1416 D
Inn. circ. = 3.1416 H

To find the area of a flat-oval—Fig. 54. Multiply the length by the width and subtract .214 times the square of the width from the result.

To find the circumference of a flat-oval. The circumference of a flat-oval is equal to twice its length plus 1.142 times its width.

Flat-oval: Area = D (H—0.214D)
Circ. = 2 (H×0.571D)

To find the area of a parabola Fig. 55. Multiply the base by the height and by .667.

Parabola: Area = .667 (D × H)

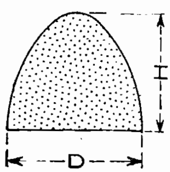

Fig. 55.

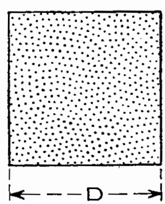

Fig. 56.

To find the area of a square—Fig. 56. Multiply the length by the width, or, in other words, the area is equal to square of the diameter.

To find the circumference of a square. The circumference of a square is equal to the sum of the lengths of the sides.

Square: Area = D^2
Circ. = 4D

To find the area of a rectangle—Fig. 57. Multiply the length by the width, the result is the area of the rectangle.

To find the circumference of a rectangle. The circumference of a rectangle is equal to twice the sum of the length and width.

Rectangle: Area $= D \times H$
Circ. $= 2\ (D \times H)$

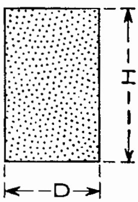

Fig. 57.

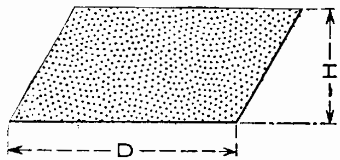

Fig. 58.

To find the area of a parallelogram Fig. 58. Multiply the base by the perpendicular height.

Parallelogram: Area $= D \times H$

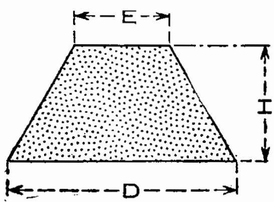

Fig. 59.

To find the area of a trapezoid—Fig. 59. Multiply half the sum of the two parallel sides by the perpendicular distance between the sides.

Trapezoid: Area $= \dfrac{(HE + D)}{2}$

To find the area of an equilateral triangle—Fig. 60. The area of an equilateral triangle is equal to the square of one side multiplied by .433.

To find the circumference of an equilateral triangle.

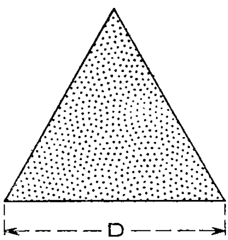

Fig. 60.

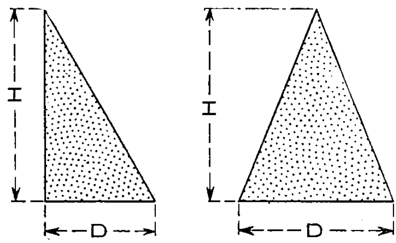

Fig. 61.

The circumference of an equilateral triangle is equal to the sum of the length of the sides.

Equilateral triangle: Area $= .433D^2$

Circ. $= 3D$

To find the area of a right-angle or an isosceles triangle—Fig. 61. Multiply the base by half the perpendicular height.

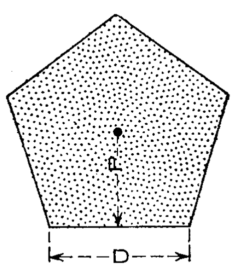

Fig. 62.

To find the circumference of any regular polygon—Fig. 62. The circumference of any polygon is equal to the sum of the length of the sides.

Polygon: $\text{Area} = \dfrac{\text{No. of sides} \times D \times P}{2}$

$\text{Circ.} = \text{No. of sides} \times D$

D = Length of one side.

P = Perdendicular distance from the center to one side.

MENSURATION OF VOLUME AND SURFACE OF SOLIDS.

To find the cubic contents of a sphere—Fig. 63. Multiply the cubic of the diameter by .5236.

To find the superficial area of a sphere. Multiply the square of the diameter by 3.1416.

Sphere: Cubic contents $= .5236D^3$

Superficial area $= 3.1416D^2$

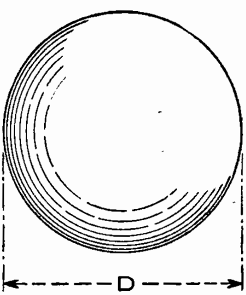

Fig. 63.

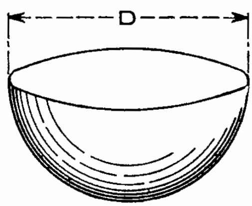

Fig. 64.

The area of the surface of a sphere is equal to the area of the surface of a cylinder, the diameter and the height of which are each equal to the diameter of the sphere. Also, the area of the surface of a sphere is equal to four times the area of its diameter.

The latter definition is easily remembered, and is useful in calculating the areas of the hemispheres, because the area of the sheet or disc of metal required for raising a hemisphere must be equal in area to the combined areas of two discs, each equal to the diameter of the hemisphere.

To find the cubic contents of a hemisphere—Fig. 64. Multiply the cube of the diameter by .2618.

To find the superficial area of a hemisphere.

Hemisphere: Cubic contents $= .2618D^3$

Superficial area $= 2.3562D^2$

To find the cubic contents of a cylindrical ring—Fig. 65. To the cross-sectional diameter of the ring add the inner diameter of the ring, multiply the sum by the

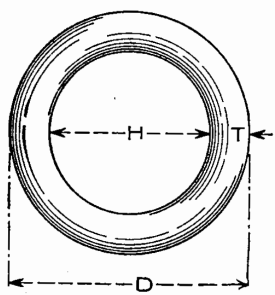

Fig. 65.

square of the cross-sectional diameter of the ring and by 2,4674. the product is the cubic contents.

To find the superficial area of a cylindrical ring. To the cross-sectional diameter of the ring add the inner diameter of the ring. Multiply the sum by the cross-sectional diameter of the ring and by 9.8696, the product is the superficial area.

Cylindrical ring: Cubic contents $= 2.4674T^2 (T + H)$
Superficial area $= 9.8696T (T + H)$
$D = (H + 2T)$

To find the cubic contents of a cylinder—Fig. 66. Multiply the area of one end by the length of the cylinder, the product will be the cubic contents of the cylinder.

To find the superficial area of a cylinder. Multiply the circumference of one end by the length of the cylinder and add to the product the area of both ends.

Cylinder: Cubic contents $= .7854 (D + H)$
Superficial area $= 1.5708D (2H + D)$

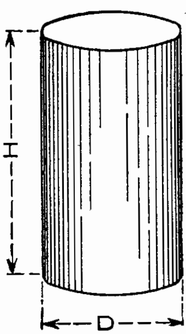

Fig. 66.

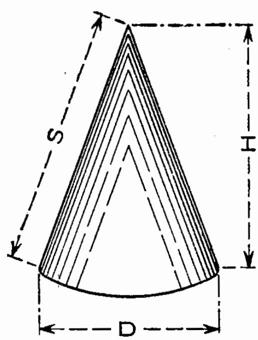

Fig. 67.

To find the cubic contents of a cone—Fig. 67. Multiply the square of the base by the perpendicular height and by .2618.

To find the superficial area of a cone. Multiply the circumference of the base by one-half the slant height and add to the product the area of the base.

Cone: Cubic contents $= .2618 (D^2 \times H)$
Superficial area $= .7854D (2S + D)$

To find the cubic contents of the frustum of a cone—Fig. 68. To the sum of the areas of the two ends of the frustum, add the square root of the product of the diameters of the two ends, this result multiplied by one-third of the perpendicular height of the frustum will give the cubic contents.

To find the superficial area of the surface of the frustum of a cone. Multiply the sum of the diameters of the ends by 3.1416 and by half the slant height. Add to the result the area of both ends and the sum of the two will be superficial area.

Frustum of cone:

$$\text{Cubic contents} = \frac{H(.2618\,(E^2+D^2)\sqrt{E \times D})}{3}$$

$$\text{Superficial area} = 3.1416S\left(\frac{D+E}{2}\right) + .7854(E^2+D^2)$$

$$S = \sqrt{\left(\frac{D-E}{2}\right)^2 + H^2}$$

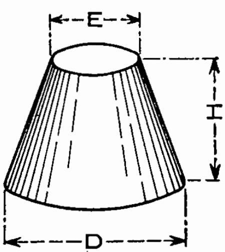

Fig. 68.

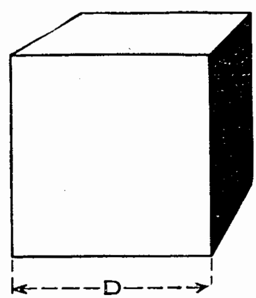

Fig. 69.

To find the contents of a cube—Fig. 69. The contents are equal to the cube of its diameter.

To find the superficial area of a cube. The superficial area of a cube is equal to six times the square of its diameter.

Cube: Cubic contents $= D^3$
Superficial area $= 6D^2$

To find the cubic contents of a rectangular solid—Fig. 70. Multiplying together the length, width and height will give the cubic contents of the rectangular solid.

To find the superficial area of a rectangular solid. Multiply the width by the sum of the height and length and add to it the product of the height multiplied by the length, twice this sum is the superficial area of the rectangular solid.

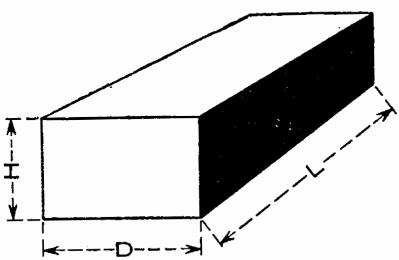

Fig. 70.

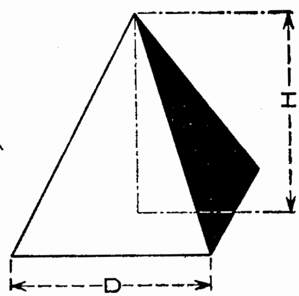

Fig. 71.

Rectangular solid:

Cubic contents $= D \times H \times L$
Superficial area $= 2\,(D\,(H + L) + HL)$

To find the cubic contents of a pyramid—Fig. 71. Multiply the area of the base by one-third the perpendicular height and the product will be the cubic contents of the pyramid.

To find the superficial area of a pyramid. Multiply the circumference of the base by half the slant height and to this add the area of the base, the sum will be the superficial area.

$$\text{Pyramid: Cubic contents} = \frac{D^2 \times H}{3}$$

$$\text{Superficial area} = \left(\frac{4D+S}{2} + 4D\right)$$

$$S = \sqrt{\frac{D^2}{4} + H^2}$$

MENSURATION OF TRIANGLES.

To find the base of a right-angle triangle when the perpendicular and the hypothenuse are given—Fig. 72.

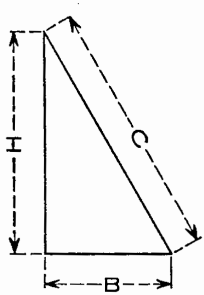

Fig. 72.

Subtract the square of the perpendicular from the square of the hypothenuse, the square root of the difference is equal to the length of the base.

$$\text{Base} = \sqrt{\text{Hypotenuse}^2 - \text{Perpendicular}^2} \text{ or } B = \sqrt{C^2 - H^2}$$

To find the perpendicular of a right-angle triangle

when the base and hypothenuse are given. Subtract the square of the base from the square of the hypothenuse, the square root of the difference is equal to the length of the perpendicular.

$$\text{Perpendicular} = \sqrt{\text{Hypotenuse}^2 - \text{Base}^2} \text{ or } H = \sqrt{C^2 - B^2}$$

To find the hypothenuse of a right-angle triangle when the base and the perpendicular are given. The square root of the sum of the squares of the base and the perpendicular is equal to the length of the hypothenuse.

$$\text{Hypotenuse} = \sqrt{\text{Base}^2 + \text{Perpendicular}^2}$$

$$C = \sqrt{B^2 + H^2}$$

To find the perpendicular height of any oblique angled triangle—Fig. 73. From half the sum of the three sides of the triangle, subtract each side severally. Multiply the half sum and the three remainders to-

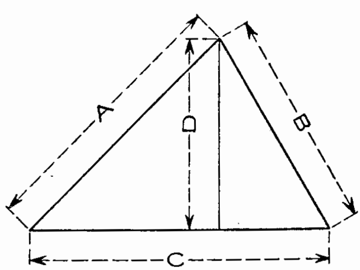

Fig. 73.

gether and twice the square root of the result divided by the base of the triangle will be the height of the perpendicular.

$$D=\frac{2\sqrt{S(S-A)\ (S-B)\ (S-C)}}{C}$$

$$S=\frac{\text{Sum of sides}}{2}$$

To find the area of any oblique angled triangle when only the three sides are given. From half the sum of the three sides, subtract each side severally. Multiply the half sum and the three remainders together and the square root of the product is equal to the area required.

$$\text{Area}=\sqrt{S(S-A)\ (S-B)\ (S-C)}$$

To find the height of the perpendicular and the two sides of any triangle inscribed in a semi-circle, when

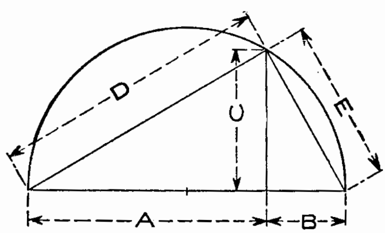

Fig. 74.

the base of the triangle and the location of the perpendicular are given—Fig. 74.

$$A=\frac{C^2}{B} \qquad B=\frac{C^2}{A} \qquad C=\sqrt{A\times B}$$

$$D=\sqrt{A(A+B)} \quad E=\sqrt{B(A+B)}$$